



# Hatherleigh CP School



Maths  
Calculation Booklet  
Revised Aut '13

+

Addition

# PROGRESSION THROUGH CALCULATIONS FOR ADDITION

## MENTAL CALCULATIONS

These are a **selection** of mental calculation strategies and should be ongoing:

### Mental recall of number bonds

$$6 + 4 = 10$$

$$25 + 75 = 100$$

$$34 + 6 = 40$$

$$0.7 + 0.3 = 1$$

$$\square + 3 = 10$$

$$19 + \square = 20$$

$$68 + \square = 70$$

$$0.07 + 0.03 = 0.1$$

Children use their knowledge of number bonds to ten to derive addition facts to 20 and then pairs of numbers that total 100. They should also explore different ways of making other numbers (e.g.  $1 + 6 = 7$ ;  $2 + 5 = 7$ ;  $3 + 4 = 7$  etc.)

### Use near doubles

$$6 + 7 = \text{double } 6 + 1 = 13$$

Children need to practise doubling numbers to 10 so that they can apply this knowledge when recognising near doubles.

### Addition using partitioning and recombining

$$34 + 45 = \quad 30 + 40 = 70$$

$$4 + 5 = 9$$

$$70 + 9 = 79$$

Partitioning is a key skill that children use for all number operations. It requires an understanding of place value (what each digit in a number is worth) and needs to be taught as a skill so that it can then be applied.

### Counting on or back in repeated steps of 1, 10, 100, 1000, 0.1, 0.01

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

### Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

These are known as compensation methods. Some children will find them really useful but be aware that they may confuse others.

### Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

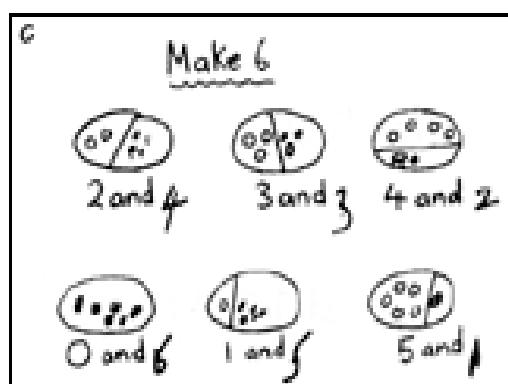
*MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED.  
THEY ARE NOT REPLACED BY WRITTEN METHODS.*

## First Stage

Children are given a lot of practical experience with a range of materials. These can include numicon, number bars, drawings or simple objects to count. The main aim is to give children a secure and concrete understanding of the process of addition, what is physically happening when they add numbers together. Fingers play a great part in this process helping children to secure their knowledge.

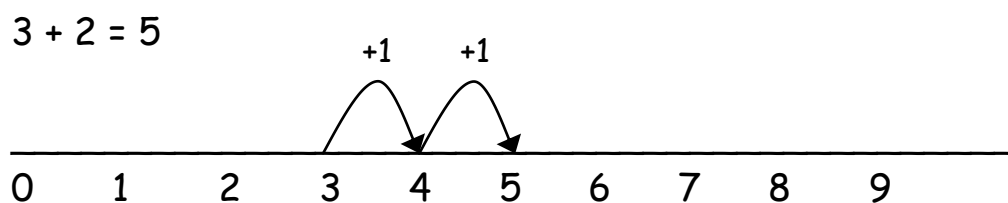
## Second Stage - Developing Written Recording

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



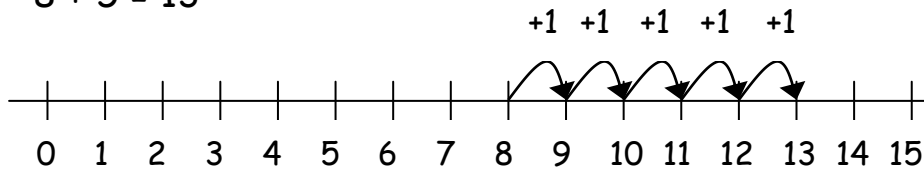
Children use number lines as a first step in moving towards more formal written methods but still use concrete objects and pictorial representations alongside to ensure they still visualise the process.

Children use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line. e.g. to add 3 and 2 they must start at three on the number line and make two jumps of one forward.

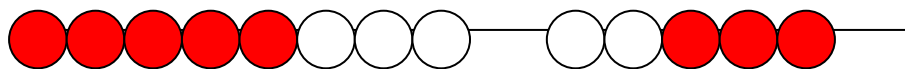


Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$$8 + 5 = 13$$



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



When they bridge through ten children are making use of their knowledge of number bonds. It is really important that these basic skills are still practised and that children are aware of when they are using them.

- ✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square + 1 = 4$$

$$20 - \triangle = 4$$

$$\square + \triangle = 14$$

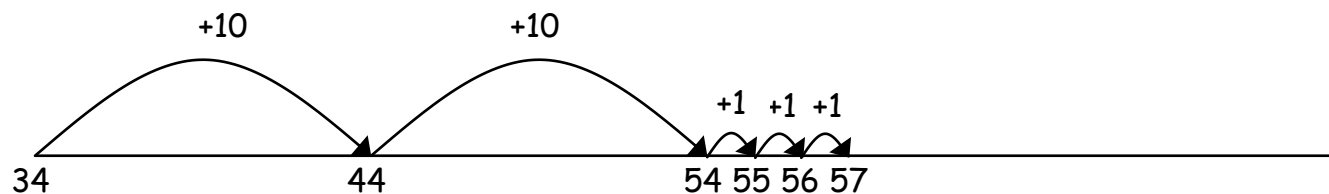
$$12 = \square + \triangle$$

### Third Stage - Using Empty/Blank Number Lines

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

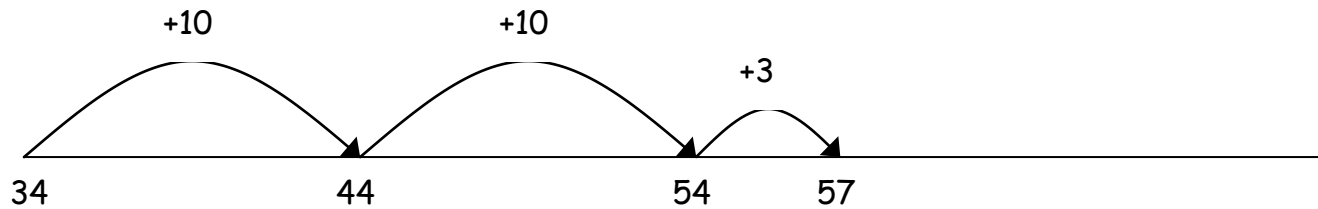
- ✓ First counting on in tens and ones.

$$34 + 23 = 57$$



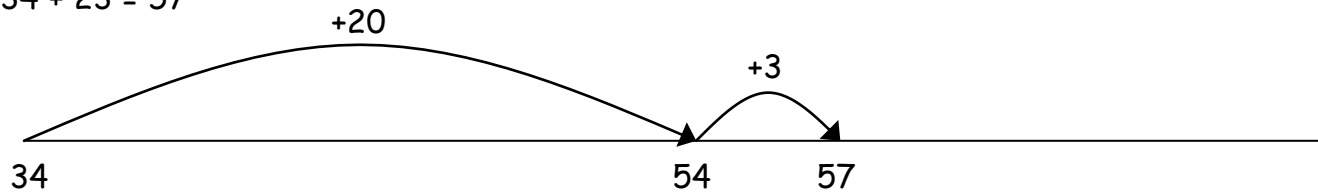
- ✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact  $4 + 3 = 7$ ). Children should continue to practise their known facts so that they can apply them quickly and easily to help with larger calculations.

$$34 + 23 = 57$$



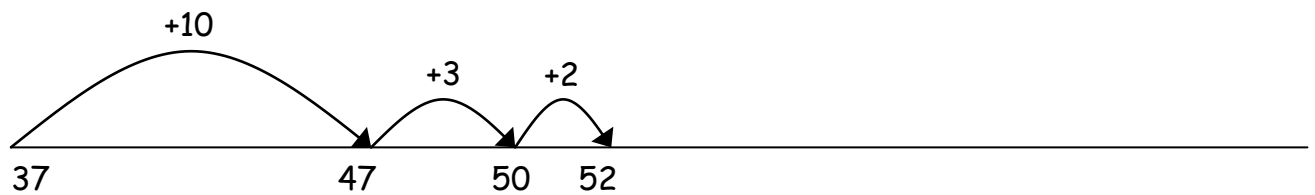
- ✓ Followed by adding the tens in one jump and the units in one jump.

$$34 + 23 = 57$$

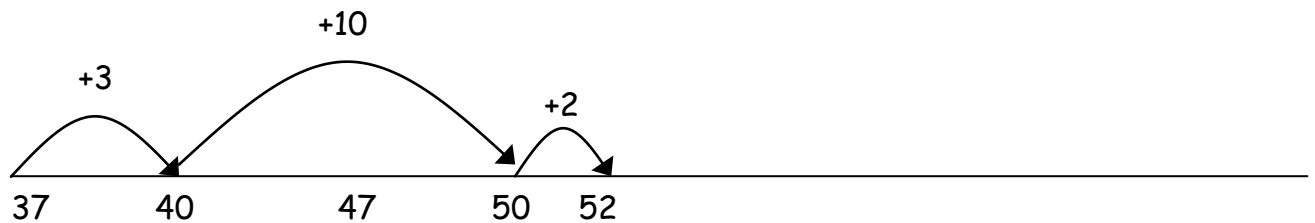


- ✓ Bridging through ten can help children become more efficient. Again, this requires children to apply their knowledge of number bonds.

$$37 + 15 = 52$$



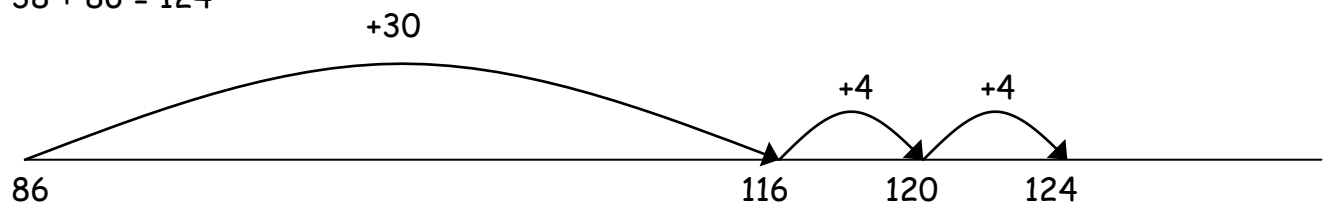
Alternatively



Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

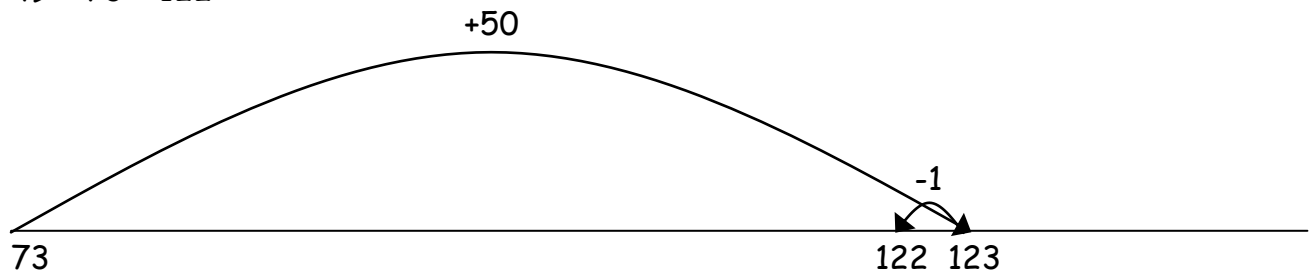
- ✓ Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$



✓ Compensation

$$49 + 73 = 122$$



Compensation methods should always be introduced using a number line so that children can see exactly what is happening. Some children will find these methods really useful, others will find them confusing.

Do not persist if the child is getting muddled.

### **Fourth Stage - Informal Pencil and Paper Methods**

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Expanded methods:

$$67 + 24 =$$

$$\begin{array}{r} 60 + 7 \\ + 20 + 4 \\ \hline 80 + 11 = 91 \end{array}$$

Lead on to:

$$\begin{array}{r} 67 \\ + 24 \\ \hline 11 \text{ ( } 7 + 4 \text{)} \\ \hline 80 \text{ ( } 60 + 20 \text{)} \\ \hline 91 \end{array}$$

$$\begin{array}{r} 267 \\ + 85 \\ \hline 12 \text{ ( } 7 + 5 \text{)} \\ 140 \text{ ( } 60 + 80 \text{)} \\ \hline 200 \\ \hline 352 \end{array}$$

The least significant digit (ones in this case) should always be added first to avoid confusion when children move on to formal written methods.

## Fifth Stage - Formal Written Methods

From this, children will begin to carry below the line.

$$\begin{array}{r} 625 \\ + 48 \\ \hline 673 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 783 \\ + 42 \\ \hline 825 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ \hline 11 \end{array}$$

*Using similar methods, children will:*

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*
- ✓ *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*

Children should extend the carrying method to numbers with at least four digits.

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \hline 111 \end{array}$$

*Using similar methods, children will:*

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m - 280 cm.*

Children should extend the carrying method to add with any number of digits.

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ \hline 111 \end{array}$$

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ \hline 111 \end{array}$$

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ \hline 121 \end{array}$$



*Using similar methods, children will*

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g.  $401.2 + 26.85 + 0.71$ .*

**By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.**

**Children should not be made to go onto the next stage if:**

- 1) they are not ready.
- 2) they are not confident.

**Children should be encouraged to approximate their answers before calculating.**

**Children should be encouraged to check their answers after calculation using an appropriate strategy.**

**Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.**

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Subtraction

# PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

## MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

### Mental recall of addition and subtraction facts

$$10 - 6 = 4$$

$$17 - \square = 11$$

$$20 - 17 = 3$$

$$10 - \square = 2$$

### Find a small difference by counting up

$$82 - 79 = 3$$

$$(79 + 3 = 82)$$

### Counting on or back in repeated steps of 1, 10, 100, 1000, 0.1, 0.01

$$86 - 52 = 34 \text{ (by counting back in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

### Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

(some children do find this confusing as they need to remember to add or subtract at the end)

### Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

*MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED.  
THEY ARE NOT REPLACED BY WRITTEN METHODS.*

## First Stage

Children are given a lot of practical experience with a range of materials. These can include numicon, number bars, drawings or simple objects to count. The main aim is to give children a secure and concrete understanding of the process of subtraction, what is physically happening when they take numbers away from each other. Fingers play a great part in this process helping children to secure their knowledge.

## Second Stage

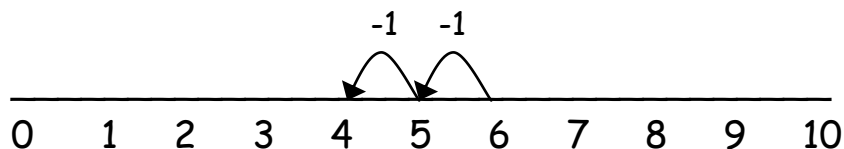
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. After much experience with a variety of materials they develop ways of recording calculations using pictures etc. Children are encouraged to think of stories to explain their number sentences.



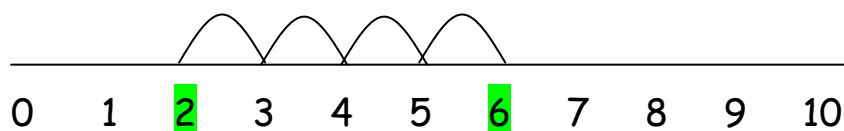
Children use number lines as a first step in moving towards more formal written methods but still use concrete objects and pictorial representations alongside to ensure they still visualise the process.

They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the numberline.

$$6 - 2 = 4$$



The numberline should also be used to show that  $6 - 2$  means the 'difference between 6 and 2' or 'the difference between 2 and 6' and how many jumps they are apart.



It is really important that children develop an awareness that subtraction is not always 'taking away'.

Jane has 6 sweets. She gives 2 sweets to Emma. How many sweets does Jane have left?

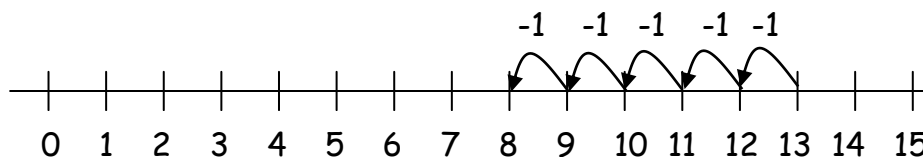
Jane has 6 sweets and Emma has 2 sweets. How many more sweets does Jane have than Emma?

Both of these situations can be written as  $6 - 2 = 4$  but one is 'taking away' and the other is 'finding the difference'.

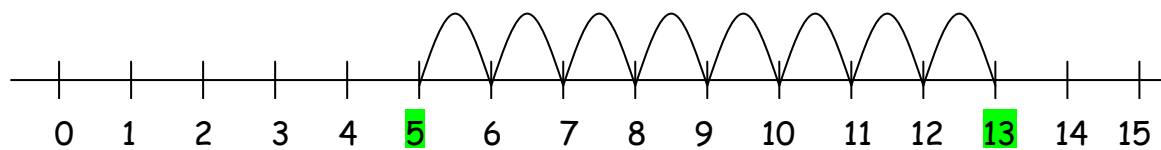
More practical equipment would also be used alongside to illustrate what happens when you take away 2 from 6 or when you compare the difference between 2 and 6.

Children then begin to use number lines to support their own calculations using a number line to count back in ones.

$$13 - 5 = 8$$

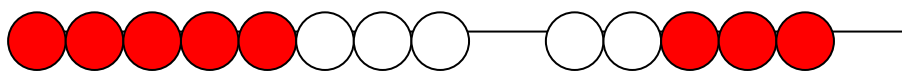


Or to find the difference in ones



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



This use of number facts is really important. Children need quick recall of number bonds to 10 and addition and subtraction facts for numbers up to 20 so that they can use them when they calculate.

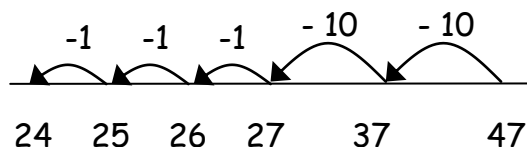
## Second Stage

Children will begin to use empty number lines to support calculations.

### Counting back

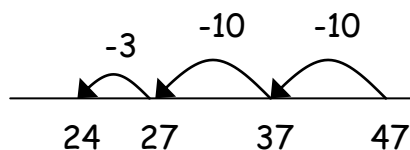
- ✓ First counting back in tens and ones.

$$47 - 23 = 24$$



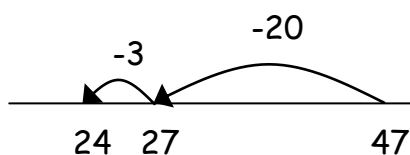
- ✓ Then helping children to become more efficient by subtracting the units in one jump (by using the **known fact**  $7 - 3 = 4$ ).

$$47 - 23 = 24$$



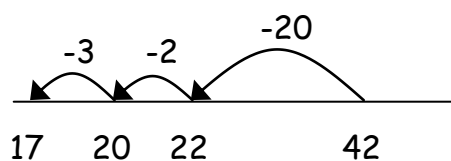
- ✓ Subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



### Counting on

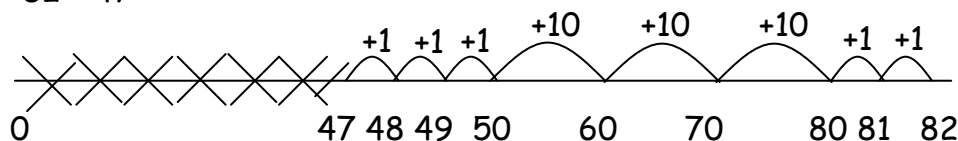
If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

For some children counting up can be confusing as they do not understand why they are 'adding' in order to 'take away'. This usually means that they do not have a clear understanding of subtraction as 'finding the difference' where nothing is taken away.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

$$82 - 47$$



**Help children to become more efficient with counting on by:**

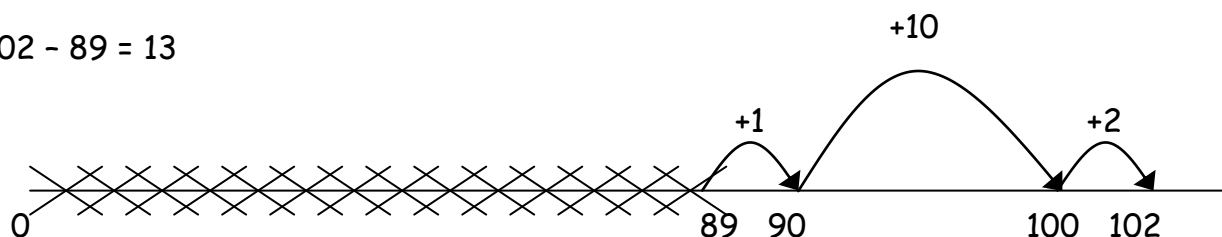
- ✓ Counting on the units in one jump;
- ✓ Counting on the tens in one jump and the units in one jump;
- ✓ Bridging through ten.

### Third Stage

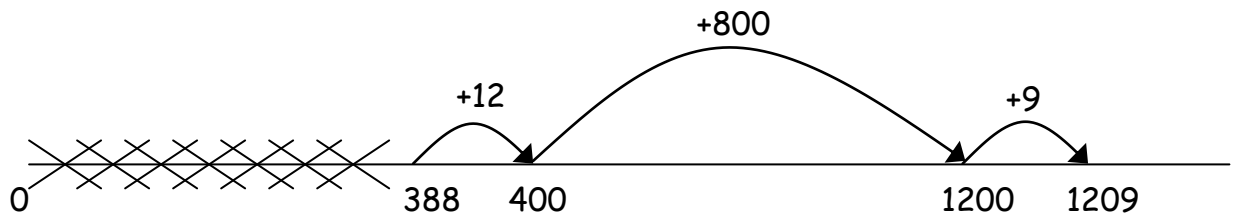
Children will continue to use empty number lines with increasingly large numbers.

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

$$102 - 89 = 13$$



$$1209 - 388 = 821$$



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

### Partitioning and decomposition

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

**NOTE** When solving the calculation  $89 - 57$ , children should know that 57 **does NOT EXIST AS AN AMOUNT** it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89.

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 + 9 \\ 50 + 7 \\ \hline 30 + 2 = 32 \end{array}$$

*Initially, the children will be taught using examples that do not need them to exchange.*

**From this the children will begin to exchange.**

$$\text{Step 1} \quad \begin{array}{r} 71 \\ - 46 \\ \hline \end{array} = \begin{array}{r} 70 + 1 \\ - 40 + 6 \\ \hline \end{array}$$

$$\text{Step 2} \quad \begin{array}{r} 60 + 11 \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$

The calculation should be read as e.g. take 6 from 1.

This would be recorded by the children as

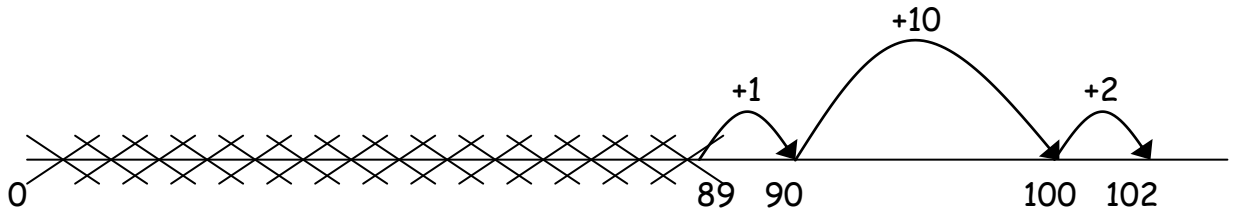
$$\begin{array}{r} \overset{60}{\cancel{70}} + 11 \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$



Children should know that units line up under units, tens under tens, and so on.

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$102 - 89 = 13$$



### Fourth Stage

Partitioning and decomposition HTU - TU

$$\text{Step 1} \quad \begin{array}{r} 754 \\ - 86 \\ \hline \end{array} = \begin{array}{r} 700 + 50 + 4 \\ - \quad \quad 80 + 6 \\ \hline \end{array}$$

$$\text{Step 2} \quad \begin{array}{r} 700 + 40 + 14 \\ - \quad \quad 80 + 6 \\ \hline \end{array} \quad (\text{adjust from } T \text{ to } U)$$

$$\text{Step 3} \quad \begin{array}{r} 600 + 140 + 14 \\ - \quad \quad 80 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array} \quad (\text{adjust from } H \text{ to } T)$$

This would be recorded by the children as

$$\begin{array}{r} \overset{600}{\cancel{700}} + \overset{140}{\cancel{50}} + 14 \\ - \quad \quad 80 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array}$$

## Decomposition

$$\begin{array}{r} 614 \text{ 1} \\ 7\cancel{8}4 \\ - \underline{86} \\ \hline 668 \end{array}$$

Children should:

- ✓ be able to subtract numbers with different numbers of digits;
- ✓ using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;
- ✓ know that decimal points should line up under each other.

For example:

$$\begin{array}{r} \text{£}8.95 = 8 + 0.9 + 0.05 \\ \text{leading to} \\ \underline{-\text{£}4.38} \quad - \underline{4 + 0.3 + 0.08} \\ \\ = \begin{array}{r} 8 + 0.8 + 0.15 \\ - 4 + 0.3 + 0.08 \\ \hline 4 + 0.5 + 0.07 \end{array} \quad (\text{adjust from T to U}) \end{array} \quad \begin{array}{r} 1 \\ 8.85 \\ - \underline{4.38} \end{array}$$

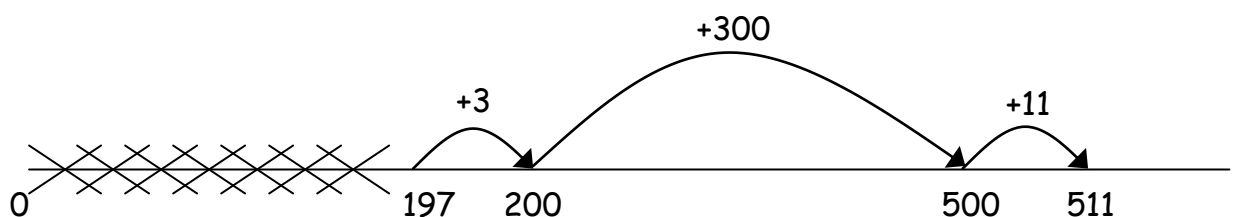
$$= \text{£}4.57$$

Alternatively, children can set the amounts to whole numbers, i.e. 895 - 438 and convert to pounds after the calculation.

**NB** If your children have reached the concise stage they will then continue this method. They will not go back to using the expanded methods.

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$511 - 197 = 314$$



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

x

Multiplication

# PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

## MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

### **Doubling and halving**

Applying the knowledge of doubles and halves to known facts.

e.g.  $8 \times 4$  is double  $4 \times 4$

### **Using multiplication facts**

*Reception and Year 1 practise counting in different amounts. From Yr 2 onwards tables should be taught, either as part of the mental oral starter or other times as appropriate within the day.*

Year R &      2 times table  
Year 1        5 times table  
                  10 times table (each taught using a range of practical materials.)

Year 2        2 times table  
                  5 times table  
                  10 times table  
                  3 times table (Summer Term)  
                  4 times table (Summer Term)

Year 3        2 times table  
                  3 times table  
                  4 times table  
                  5 times table  
                  6 times table  
                  10 times table

Year 4        Derive and recall all multiplication facts up to  $12 \times 12$

Years 5 & 6 Derive and recall quickly all multiplication facts up to  $12 \times 12$ .

### **Multiplying by 10 or 100**

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

### Using and applying multiplication and division facts

Children should be able to utilise their tables knowledge to derive other facts, for example, by using knowledge of place value, equivalent facts and near facts. e.g. If I know  $3 \times 7 = 21$ , what else do I know?

$30 \times 7 = 210$ ,  $300 \times 7 = 2100$ ,  $3000 \times 7 = 21\ 000$ ,  $0.3 \times 7 = 2.1$ ,  $1.5 \times 14 = 21$ ,  $6 \times 3.5 = 21$ ,  $4 \times 7 = 28$  etc.

### Use closely related facts already known

$$\begin{aligned} 13 \times 11 &= (13 \times 10) + (13 \times 1) \\ &= 130 + 13 \\ &= 143 \end{aligned}$$

### Partitioning numbers in different ways (not always into tens and ones)

$$\begin{aligned} 23 \times 4 &= (20 \times 4) + (3 \times 4) \\ &= 80 + 12 \\ &= 92 \end{aligned}$$

$$\begin{aligned} 8 \times 7 &= (8 \times 2) + (8 \times 5) \\ &= 16 + 40 \\ &= 56 \end{aligned}$$

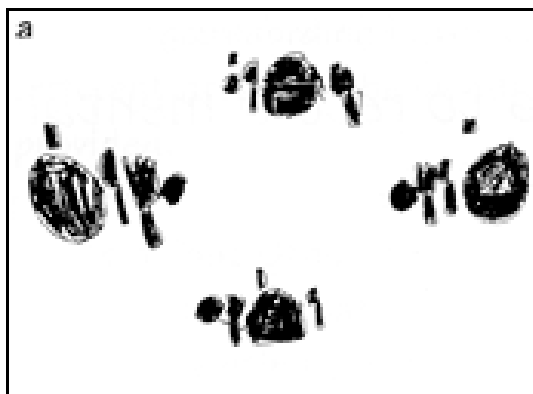
### Use of factors

$$8 \times 12 = 8 \times 4 \times 3$$

*MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.*

### First Stage

Children learn to sort objects in a variety of ways through looking for likenesses. They make repeating patterns with colour/shape/objects, then sets of numbers. Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.



## Second Stage

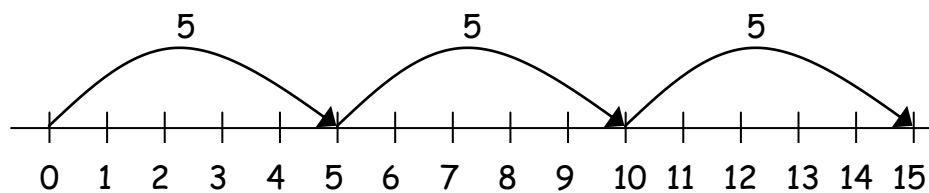
Children will develop their understanding of multiplication and use jottings to support calculation:

### ✓ Repeated addition

5 times 3 is  $5 + 5 + 5 = 15$  or 3 lots of 5 or  $5 \times 3$  (5, three times)

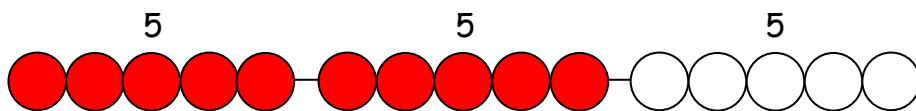
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



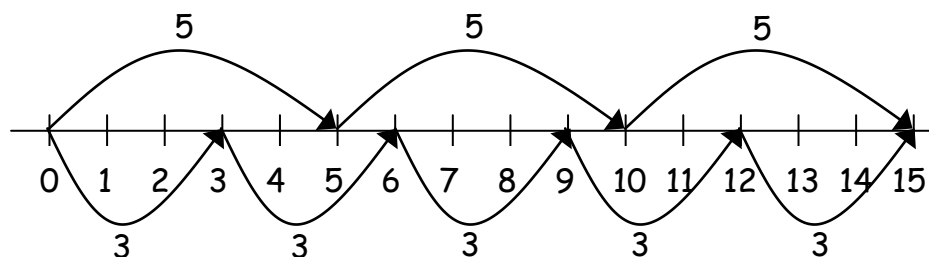
and on a bead bar:

$$5 \times 3 = 5 + 5 + 5$$



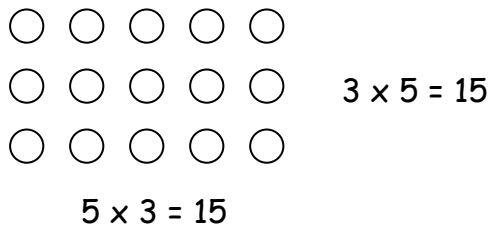
### ✓ Commutativity

Children should know that  $3 \times 5$  has the same answer as  $5 \times 3$ . This can also be shown on the number line.



✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



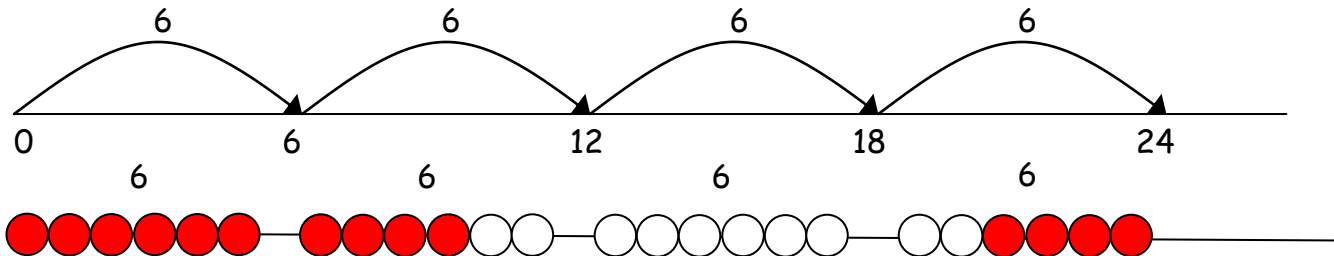
**Third Stage**

Children will continue to use:

✓ **Repeated addition**

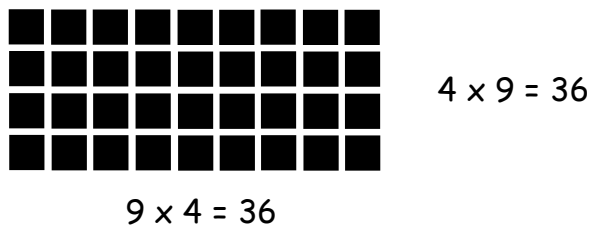
4 times 6 is  $6 + 6 + 6 + 6 = 24$  or 4 lots of 6 or  $6 \times 4$

Children should use number lines or bead bars to support their understanding.



✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.





They should explore how the array can be split in different ways through using their knowledge of number facts. E.g.

$$4 \times 9 = (4 \times 1) + (4 \times 1) + (4 \times 1) + (4 \times 1) + (4 \times 1) + (4 \times 1) + (4 \times 1) + (4 \times 1) + (4 \times 1) \text{ or}$$

$$4 \times 9 = (4 \times 2) + (4 \times 7) \text{ or}$$

$$4 \times 9 = (4 \times 3) + (4 \times 6) \text{ or}$$

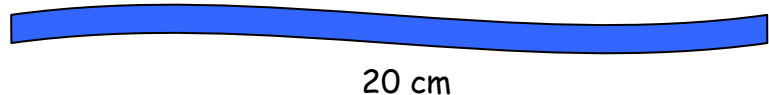
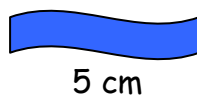
$$4 \times 9 = (4 \times 4) + (4 \times 5) \text{ or}$$

$$4 \times 9 = (4 \times 2) + (4 \times 2) + (4 \times 5) \text{ etc.}$$

Children will also develop an understanding of

✓ **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon



✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times \circ = 32$$

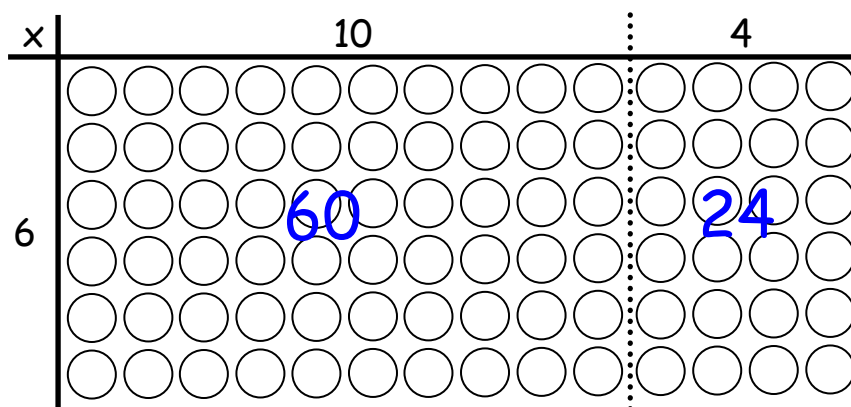
✓ **Partitioning**

$$\begin{aligned} 38 \times 5 &= (30 \times 5) + (8 \times 5) \\ &= 150 + 40 \\ &= 190 \end{aligned}$$

They will continue to make use of known multiplication facts and place value to help break calculations down.

**Fourth Stage**

Children will continue to use arrays where appropriate leading into the grid method of multiplication.



$$\begin{aligned} &(6 \times 10) + (6 \times 4) \\ &60 + 24 \\ &84 \end{aligned}$$

### Grid method

Short multiplication - multiplication by a single digit

**TU x U**

$$23 \times 8$$

Children will approximate first

$$23 \times 8 \text{ is approximately } 25 \times 8 = 200$$

$$\begin{array}{r} \times \quad 20 \quad 3 \\ 8 \quad \boxed{160} \quad \boxed{24} \end{array}$$

$$\begin{array}{r} 160 \\ + \quad 24 \\ \hline 184 \end{array}$$

**HTU x U**

$$346 \times 9$$

Children will approximate first

$$346 \times 9 \text{ is approximately } 350 \times 10 = 3500$$

$$\begin{array}{r} \times \quad 300 \quad 40 \quad 6 \\ 9 \quad \boxed{2700} \quad \boxed{360} \quad \boxed{54} \end{array}$$

$$\begin{array}{r} 2700 \\ + \quad 360 \\ + \quad 54 \\ \hline 3114 \\ \quad 11 \end{array}$$

Also

$$\begin{array}{r} 346 \\ \times 9 \\ \hline 54 \\ 360 \\ 2700 \\ \hline 3114 \\ \quad 11 \end{array}$$

## Fifth Stage

Children progress to long multiplication - multiplication by more than a single digit.

**TU x TU**

$$72 \times 38$$

Children will approximate first

$$72 \times 38 \text{ is approximately } 70 \times 40 = 2800$$

x	70	2	
30	2100	60	2100
8	560	16	+ 560
			+ 60
			+ <u>16</u>
			<u>2736</u>
			1

*Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.*

e.g.  $4.9 \times 3$

Children will approximate first

$$4.9 \times 3 \text{ is approximately } 5 \times 3 = 15$$

x	4	0.9	
3	12	2.7	12
			+ <u>2.7</u>
			<u>14.7</u>

## Sixth Stage

ThHTU × U

$$4346 \times 8$$

Children will approximate first

$$4346 \times 8 \text{ is approximately } 4346 \times 10 = 43460$$

x	4000	300	40	6
8	32000	2400	320	48

  
$$\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$$

Also at this stage, when multiplying by a single digit:

$$\begin{array}{r} 4346 \\ \times \quad 8 \\ \hline 48 \\ 320 \\ 2400 \\ \underline{32000} \\ \underline{34768} \end{array}$$

Then using a more compact method:

$$\begin{array}{r} 4346 \\ \times \quad 8 \\ \hline 34768 \\ \quad 2 \quad 3 \quad 4 \end{array}$$

## HTU x TU

$$372 \times 24$$

Children will approximate first

$372 \times 24$  is approximately  $400 \times 25 = 10000$

x	300	70	2
20	6000	1400	40
4	1200	280	8

$$\begin{array}{r} 6000 \\ + 1400 \\ + 1200 \\ + 280 \\ + 40 \\ + 8 \\ \hline 8928 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 372 \\ \times 24 \\ \hline 6000 \\ 1400 \\ 40 \\ 1200 \\ 280 \\ 8 \\ \hline 8928 \\ \hline 1 \end{array}$$

Then using more compact methods

$$\begin{array}{r} 372 \\ \times 24 \\ \hline 7440 \\ 1488 \\ \hline 8928 \\ \hline 1 \end{array}$$

*Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.*

*For example:*

$$4.92 \times 3$$

Children will approximate first

$$4.92 \times 3 \text{ is approximately } 5 \times 3 = 15$$

x	4	0.9	0.02	
3	12	2.7	0.06	
				12
				+ 0.7
				+ <u>0.06</u>
				<u>12.76</u>

**By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.**

**Children should not be made to go onto the next stage if:**

- 1) they are not ready.
- 2) they are not confident.

**Children should be encouraged to approximate their answers before calculating.**

**Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.**

÷

Division

# PROGRESSION THROUGH CALCULATIONS FOR DIVISION

## MENTAL CALCULATIONS

(ongoing) These are a **selection** of mental calculation strategies:

### Doubling and halving

Knowing that halving is dividing by 2

### Deriving and recalling division facts

*Reception and Year 1 practise counting in different amounts. From Yr 2 onwards tables should be taught, either as part of the mental oral starter or other times as appropriate within the day.*

Year R &    2 times table  
Year 1       5 times table  
              10 times table (each taught using a range of practical materials.)

Year 2       2 times table  
              5 times table  
              10 times table  
              3 times table (Summer Term)  
              4 times table (Summer Term)

Year 3       2 times table  
              3 times table  
              4 times table  
              5 times table  
              6 times table  
              10 times table

Year 4       Derive and recall division facts for all tables up to  $12 \times 12$

Year 5 & 6   Derive and recall quickly division facts for all tables up to  $12 \times 12$

### Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts, for example, by using knowledge of place value, equivalent facts and near facts.

e.g. If I know  $3 \times 7 = 21$ , what else do I know?

$30 \times 7 = 210$ ,  $300 \times 7 = 2100$ ,  $3000 \times 7 = 21\ 000$ ,  $0.3 \times 7 = 2.1$ ,  $1.5 \times 14 = 21$ ,  $6 \times 3.5 = 21$ ,  $4 \times 7 = 28$ ,  $21 \div 7 = 3$ ,  $21 \div 3 = 7$ ,  $2.1 \div 7 = 0.3$



### **Dividing by 10 or 100**

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

### **Use of factors**

$$378 \div 21 \quad 378 \div 3 = 126 \quad 378 \div 21 = 18$$
$$126 \div 7 = 18$$

### **Use related facts**

Given that  $1.4 \times 1.1 = 1.54$

What is  $1.54 \div 1.4$ , or  $1.54 \div 1.1$ ?

*MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED.  
THEY ARE NOT REPLACED BY WRITTEN METHODS.*

## First Stage

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

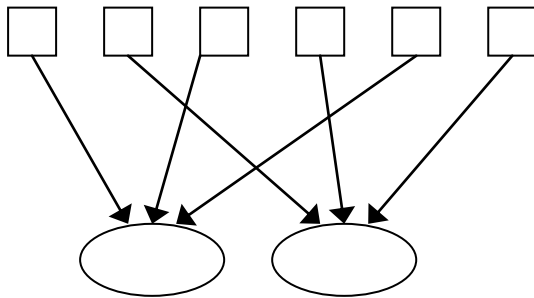


## Second Stage

Children will develop their understanding of division and use jottings to support calculation

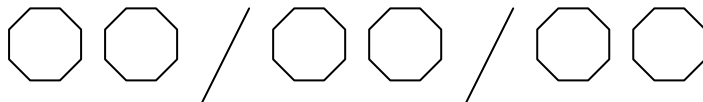
### ✓ **Sharing equally**

6 sweets shared between 2 people, how many do they each get?



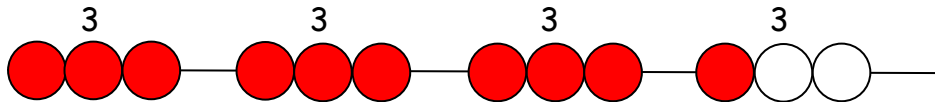
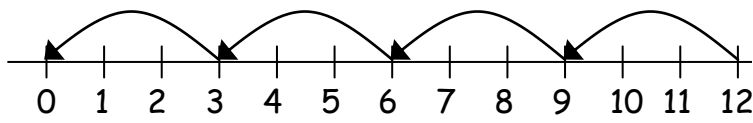
### ✓ **Grouping or repeated subtraction**

There are 6 sweets, how many people can have 2 sweets each?



- ✓ **Repeated grouping using a number line or bead bar**

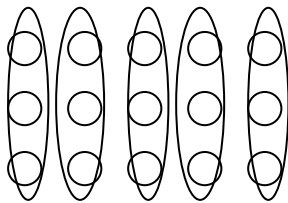
$$12 \div 3 = 4$$



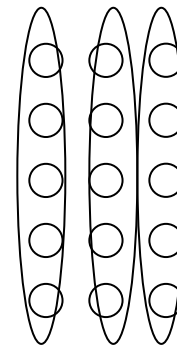
The bead bar will help children with interpreting division calculations such as  $10 \div 5$  as 'how many 5s make 10?'

- ✓ **Arrays**

Arrays should be used to develop children's understanding of the links between multiplication and division.



$$15 \div 3 = 5$$



$$15 \div 5 = 3$$

- ✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \div 2 = 4$$

$$20 \div \triangle = 4$$

$$\square \div \triangle = 4$$

$$4 = \square \div \triangle$$

### Third Stage

Ensure that the emphasis is on grouping rather than sharing.

Children will continue to use either:

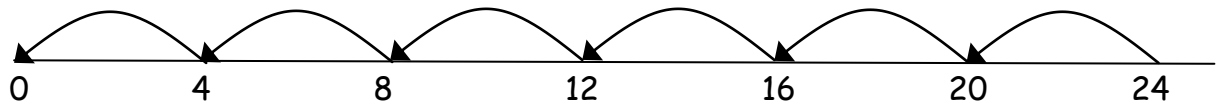
- ✓ **Repeated subtraction using a number line**

Or

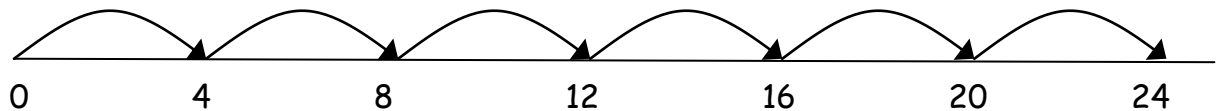
- ✓ **Repeated addition using a number line**

Children will use an empty number line to support their calculation.

$$24 \div 4 = 6$$

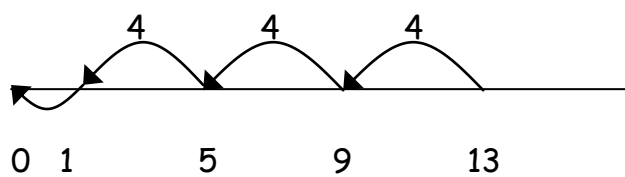


$$24 \div 4 = 6$$

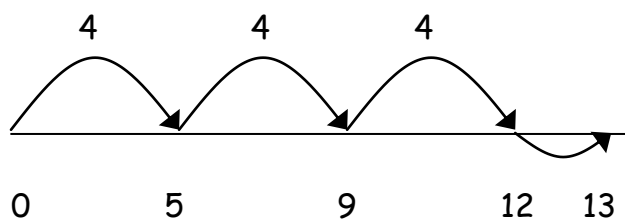


Children should also move onto calculations involving remainders.

$$13 \div 4 = 3 \text{ r } 1$$



$$13 \div 4 = 3 \text{ r } 1$$



✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$26 \div 2 = \square$$

$$24 \div \triangle = 12$$

$$\square \div 10 = 8$$

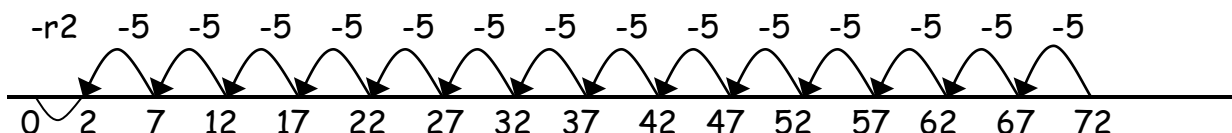
$$6 = \square \div \triangle$$

## Fourth Stage

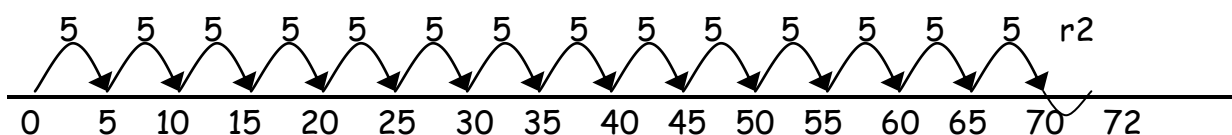
Children will develop their ability to group **bigger numbers into** multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s - numbers with which the children are more familiar.

$$72 \div 5$$

Either

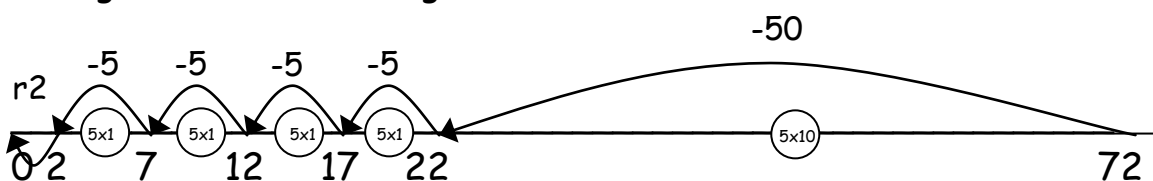


Or

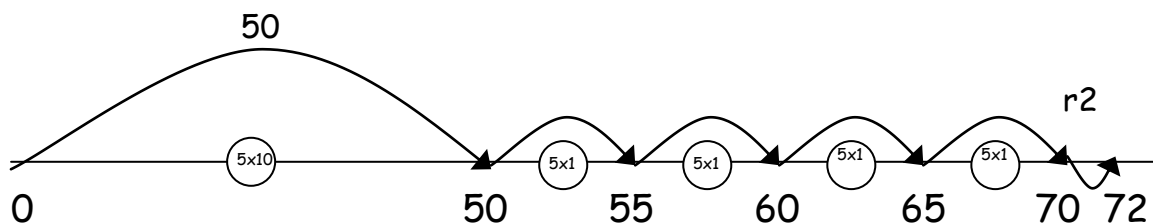


Both these examples illustrate that 72 can be split into 14 lots of  $5 \times 1$  with 2 left over.

**Moving onto either counting back:**



**Or counting on:**



While these examples illustrate that 72 can be split into 1 lot of  $5 \times 10$  and 4 lots of  $5 \times 1$  with 2 left over.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example  $62 \div 8$  is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?

Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?

Answer: 8 (the remaining 6 apples still need to be placed into a box)

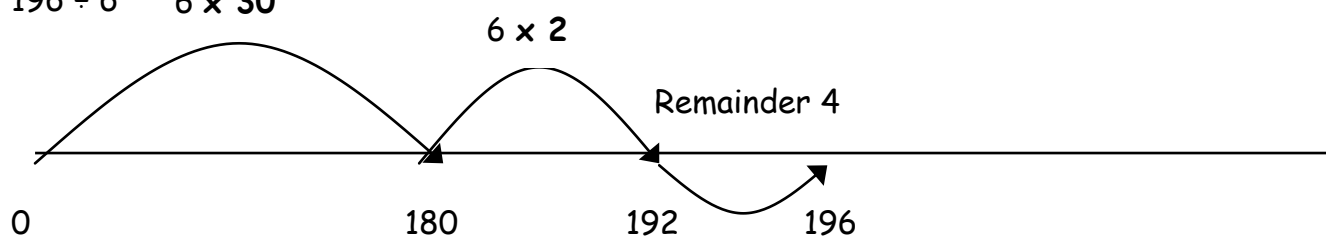
### Fifth Stage

Children will continue to use written methods to solve short division  $TU \div U$ .

Children can start to find larger multiples of the divisor, e.g.  $30 \times 6$

#### Short division $HTU \div U$

$$196 \div 6 \quad 6 \times 30$$



Answer : 32 remainder 4 or 32 r 4

Any remainders should be shown as integers, i.e. 32 remainder 4 or 32r4.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example  $240 \div 52$  is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

## Sixth Stage

Children will begin to use written methods to solve short division.

$TU \div U$  and  $HTU \div U$ .

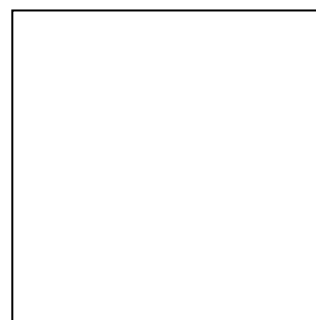
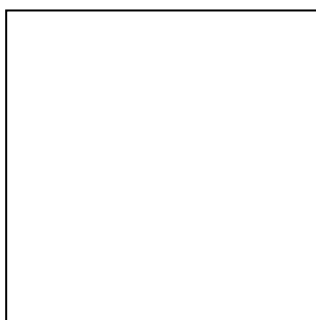
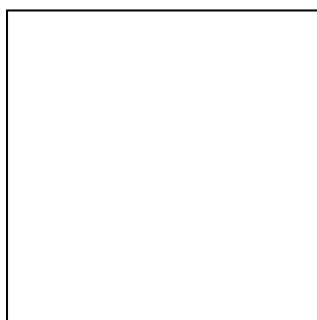
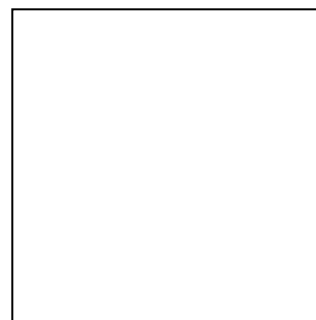
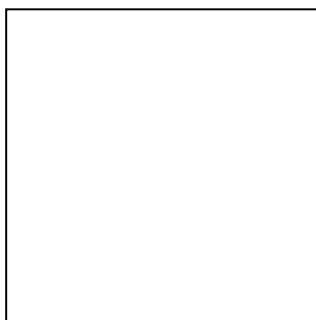
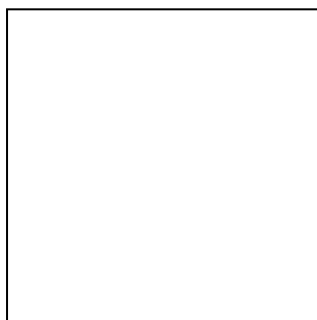
Short division deals with each digit in each column in turn and splitting the larger multiple of 10/100/1000 into the same number of groups as the divisor.

So for  $783 \div 3$

Set out like this

$$3 \overline{) 786}$$

First share the 7 hundreds between 3 groups as whole hundreds

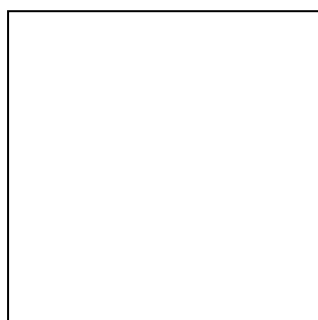


Grp 1

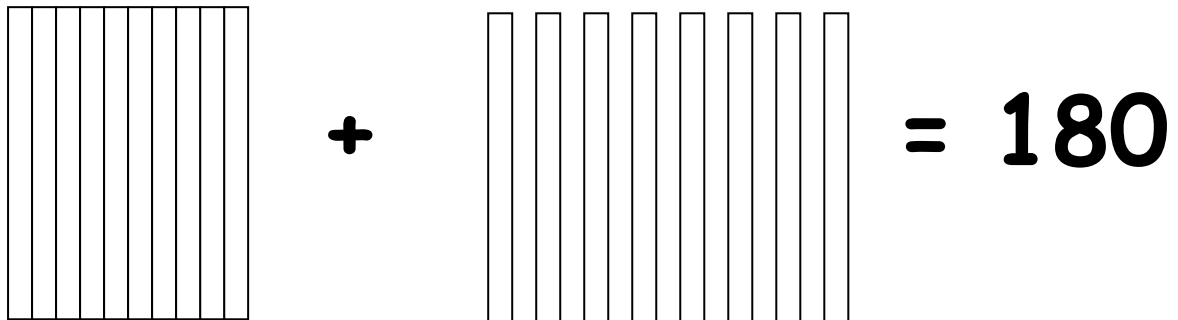
Grp 2

Grp 3

Remainder 1 lot of 100



This remaining 100 is then split into ten 10s and added onto the tens already there.

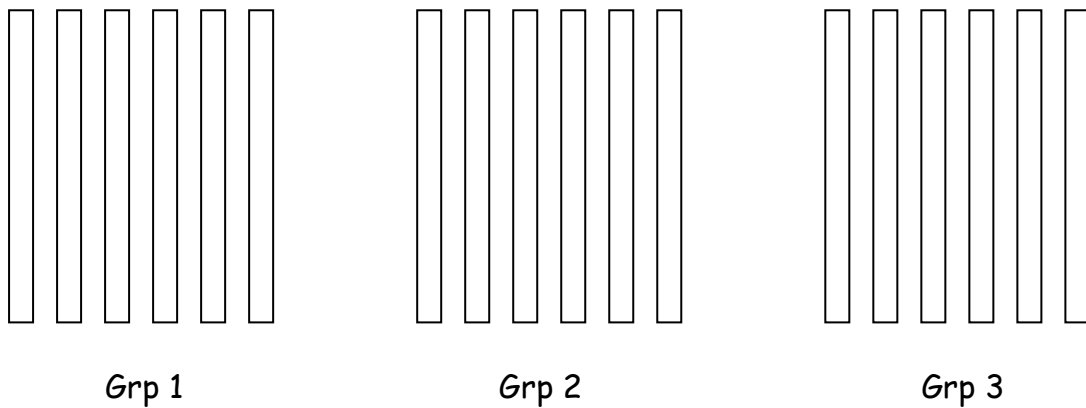


$$100 + 80 = 180$$

This is then written as;

$$\begin{array}{r} 2 \\ \hline 3 \overline{) 7 \text{ } ^1 8 \text{ } 6} \end{array}$$

The 18 tens are then shared into 3 groups.



$$\begin{array}{r} 2 \text{ } 6 \\ \hline 3 \overline{) 7 \text{ } ^1 8 \text{ } 6} \end{array}$$

The last step step is to divide the unit/ones column into 3 groups.



<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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Grp 1	Grp 2	Grp 3

$$\begin{array}{r} 262 \\ 3 \overline{) 786} \end{array}$$

### Seventh Stage

Children will continue to use written methods to solve short division  $TU \div U$  and  $HTU \div U$ , and also extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

$$87.5 \div 7$$

$$\begin{array}{r} 12.5 \\ 7 \overline{) 87.5} \end{array}$$

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.